CONJUGATE PROBLEM OF NATURAL CONVECTION IN A VERTICAL CHANNEL WITH HEAT RELEASE FROM A HIGH-FREQUENCY ELECTRIC CURRENT

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It is shown that allowance for the finite thickness and finite thermal conductivity of the walls results in more intense convection and heat exchange. The region of parameters where the problem of convective heat exchange must be stated as conjugate is indicated.

1. The convective motion of an electrically conducting liquid between parallel planes in a magnetic field has been considered in a number of works. The systematic study of this question both theoretically and experimentally began long ago [1] and continues to the present [2-5]. The statement of the problem has gradually become complicated. Reports have appeared in which the effects of the electrical conductivity of the wall, of viscous dissipation, and of the Joule heat release of a constant current on the magnetohydrodynamic flow in a flat channel are discussed [6-9]. The problem has been formulated as conjugate, i.e., the energy equations in the liquid and the walls are solved jointly using MHD equations, with the temperatures and heat fluxes being equal at the solid–liquid boundary [10-13].

In the present report the conjugate problem is examined with allowance for the Joule heat release due to the flow of a high-frequency current.

2. The one-dimensional stationary convection of an electrically conducting liquid in a vertical channel of width $2l$ with thermally conducting walls of thickness h is examined. The external magnetic field B_0 is constant and perpendicular to the walls (along the x axis). The variable electric current flows in a direction perpendicular to the plane of the stream and the magnetic field (along the z axis). It is assumed that the voltage of the applied electric field is much greater than the induced voltage $E_z \gg E_{z,\text{in}}$ and $j^2/\sigma \simeq j_z^2/\sigma$. In the fully developed one-dimensional mode all the values depend only on x (except for the pressure). Consequently the induced magnetic field has a component only along the y axis. The equations for the region of the liquid are written as follows:

$$0 = \nu \frac{d^2v}{dx^2} + \beta g (T - T_0) + \frac{1}{\rho} B_0 j_z; \tag{1}$$

$$0 = \lambda \frac{d^2T}{dx^2} + \frac{j_z^2}{\sigma}; \tag{2}$$

$$j_z = \sigma (E_z - B_0 v);$$

$$\frac{\partial^2 E_z}{\partial x^2} = \mu_0 \sigma \frac{\partial E_z}{\partial t}.$$

The heat conduction equation for the walls, in which there are no internal heat sources, will be

$$\frac{d^2T}{dx^2} = 0. \tag{3}$$

The following boundary conditions are adopted for Eqs. (1)-(3):

$$\text{for } x = \pm l \quad v = 0; \quad T = T_1; \quad \lambda \frac{dT}{dx} = \lambda_1 \frac{dT_1}{dx}; \quad E_z = E_{z0};$$
$$\text{for } x = \pm (l + h) \quad T_1 = T_0 \pm \Delta T. \tag{4}$$

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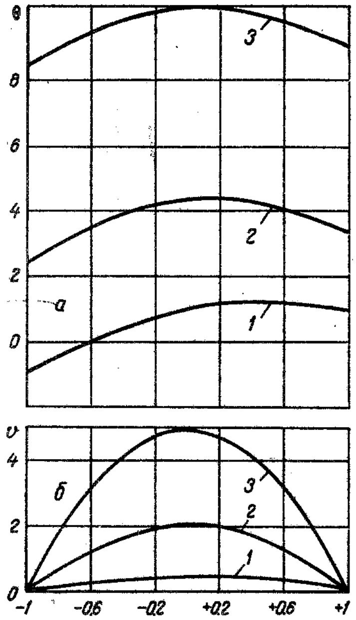

Fig. 1. Profiles of dimensionless temperature (a) and velocity (b) over channel cross section at S = 3 and n = 1 for different Ψ. Curves 1, 2, and 3 correspond to the values 0, 1, and 3 for Ψ.

Here all the notation is standard and the index 1 pertains to the walls; μ_0 is the magnetic permeability of a vacuum ($4\pi\cdot 10^{-7}$).

In order to jointly solve Eqs. (1)-(4) it is assumed that $E_z(x, t) = E_{z2}(x)\exp(i\omega t)$ and that the frequency of the electric field is high enough that the Joule heat release can be replaced by a value averaged over the period:

$$\langle E_z^2 \rangle = \frac{|E_{z2}|^2}{2}; \quad \langle E_z \rangle = 0 \quad [14].$$

Let us convert to dimensionless variables in Eqs. (1)-(4). The following are chosen as the scale for the distance, temperature, velocity, and field, respectively: l, ΔT, $V = g\beta l^2\Delta T/\nu$; E_{z20}. If one introduces the dimensionless values $\tilde{x} = x/l$; $\theta = (T-T_0)/\Delta T$; $\tilde{v} = v/V$, $\tilde{E}_{z2} = E_{z2}/E_{z20}$, then Eqs. (1)-(4) are written (omitting the sign ~) as:

$$\frac{d^2\theta}{dx^2} + S|E_{z2}|^2 + \mathrm{Ha}^2 N v^2 = 0; \tag{5}$$

$$\frac{d^2 v}{dx^2} + \theta - \mathrm{Ha}^2 v = 0; \tag{6}$$

$$\frac{d^2 E_{z2}}{dx^2} = 2in^2 E_{z2};$$

$$\frac{d^2\theta_1}{dx^2} = 0; \tag{7}$$

$$x = \pm 1: \quad v = 0; \quad \theta = \theta_1; \quad \frac{d\theta}{dx} = \frac{\lambda_1}{\lambda}\frac{d\theta_1}{dx}; \quad E_{z2} = 1;$$

$$x = \pm\left(1 + \frac{h}{l}\right): \quad \theta_1 = \pm 1. \tag{8}$$

The dimensionless parameters Ha, S, n, and N have the following form: $\mathrm{Ha} = B_0 l\sqrt{\sigma/\nu\rho}$ is the Hartmann number; $S = \sigma l^2 E_{Z20}^2/2\lambda\Delta T$ characterizes the ratio of Joule heat to the heat transmitted by thermal conduction; $n = l\sqrt{\mu_0\sigma\omega/2}$ characterizes the ratio of the half-width of the channel to the thickness of the electrical skin layer; $N = g^2 l^4 \rho\beta^2\Delta T/\lambda\nu$ characterizes the body force.

The boundary conditions for the temperature of the liquid can be written in another form. By solving Eqs. (7) with the appropriate boundary conditions (8) we will have

$$x = +1: \frac{d\theta}{dx} = \frac{1-\theta}{\Psi}; \quad x = -1: \frac{d\theta}{dx} = \frac{1+\theta}{\Psi},$$

where $\Psi = \lambda h/\lambda_1 l$ characterizes the ratio of thermal conductivities and thicknesses between the liquid and the walls. At $\Psi = 0$ we obtain the ordinary thermal boundary conditions when a constant but different temperature is given at the walls: $\theta(\pm 1) = \pm 1$.

Equations (5) are nonlinear, but since the parameter N is small one can seek a solution θ, v in the form [6, 8]

$$\theta = \theta_0 + N\theta_2, \quad v = v_0 + Nv_2,$$

where θ_0, v_0 is the solution for the case when the Joule heat release from the induced current is neglected in the energy equation; θ_2 and v_2 are disturbances relative to θ_0 and v_0.

One can clearly show when this analysis is justified. For example, for mercury when l = 1 cm we have ΔT = 5°C and $N = 2.4\cdot 10^{-3}$.

For the null approximation we obtain a system of linear equations

$$\frac{d^2\theta_0}{dx^2} + S|E_{z2}|^2 = 0, \quad \frac{d^2 v}{dx^2} - \mathrm{Ha}^2 v_0 + \theta_0 = 0$$

with the boundary conditions (8).

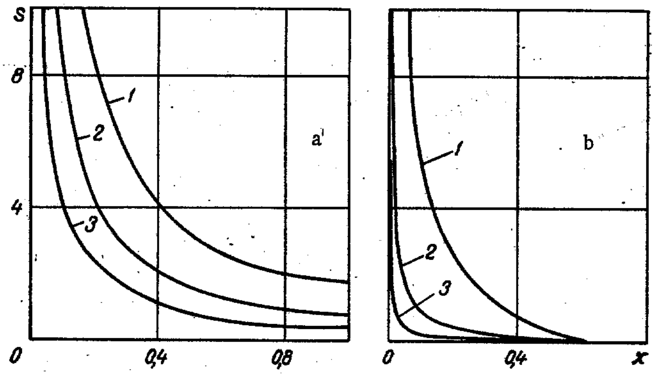

Fig. 2. Position of maximum in temperature and velocity as a function of S and Ψ at n = 1. Curves 1, 2, and 3 correspond to the values 0, 1, and 3 for Ψ.

Let us write the solution for E_{z2}, θ_0, v_0:

$$|E_{z2}|^2 = \frac{N_1(2nx)}{N_1(2n)};$$

$$\theta_0 = S\,\frac{N_2(2n) - N_2(2nx)}{4n^2 N_1(2n)} + \frac{x}{1+\Psi} + S\Psi\,\frac{\sin 2n + \operatorname{sh} 2n}{2nN_1(2n)}; \tag{9}$$

$$v_0 = A\operatorname{ch}\mathrm{Ha}\,x + B\operatorname{sh}\mathrm{Ha}\,x + \frac{S}{4n^2N_1(2n)}\left(\frac{\operatorname{ch} 2nx}{4n^2 - \mathrm{Ha}^2} + \frac{\cos 2nx}{4n^2 + \mathrm{Ha}^2}\right) - \frac{x - D}{\mathrm{Ha}^2(1+\Psi)};$$

$N_{1,2}(\xi) \equiv \operatorname{ch}\xi \pm \cos\xi$, A, B, and D are constants of integration.

The expressions for θ_2 and v_2 are omitted since they are cumbersome. The system of equations (5)-(7) can be solved numerically.

3. Let us study in detail the effect of the finite thickness and finite thermal conductivity of the walls on the convection and heat exchange in a channel without an external magnetic field and also in the case when this effect can be neglected. For this we write the definite solution which is obtained from Eqs. (5) or (9) by setting Ha = 0 there:

$$\theta \equiv \theta_0;$$

$$v = \frac{S}{16n^4N_1(2n)}\left[N_1(2nx) - N_1(2n) + 2n^2N_2(2n)(1-x^2)\right] + \frac{x - x^3}{6(1+\Psi)} + S\Psi\,\frac{\operatorname{sh} 2n + \sin 2n}{4n\,N_1(2n)}\,(1-x^2). \tag{10}$$

Let us consider some limiting cases of (10). Suppose there is no current (S = 0) and the wall thickness is infinitely small ($h \to 0$) or the thermal conductivity of the walls is infinitely large ($\lambda_1 \to \infty$), i.e., $\Psi = 0$. Then the temperature and velocity profiles represent a straight line and a cubic parabola, respectively. This is the well-studied case of natural convection in a vertical channel with a constant and different wall temperature [15].

The combined natural convection and convection induced by a high-frequency electric field in a channel occurs when $\Psi = 0$ [16]. Then the temperature and velocity profiles vary as a function of the voltage (the parameter S) and the frequency of the current (the parameter n): a maximum develops in the temperature while the minimum disappears for the velocity in the cavity.

Another limiting case is obtained from (10) when $n \to 0$. Then the heat release is the greatest and the corresponding temperature and velocity distributions have the following form:

$$\theta = \frac{S}{2}(1-x^2) + \frac{x}{1+\Psi} + S\Psi,$$

$$v = \frac{S}{4}(1-x^2) + \frac{x-x^3}{6(1+\Psi)} + S\Psi\,\frac{1-x^2}{2}. \tag{11}$$

It follows from (11) that for a fixed S the temperature and velocity increase with an increase in Ψ. This means that with the conjugate condition the heat is drawn off from the walls less than in the case of natural

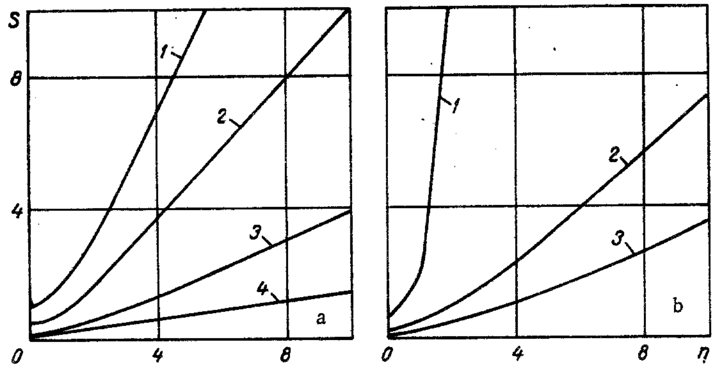

Fig. 3. Curves (1–4) for Q = 0 and Ψ = 0, 1, 5, 10 (a) and v_{min} for Ψ = 0, 0.5, 1 (b) as a function of S and n.

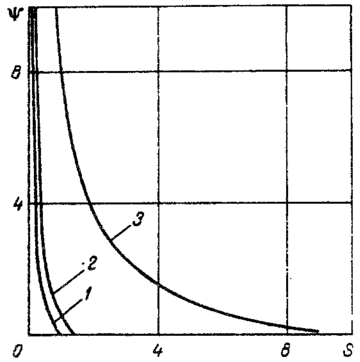

Fig. 4. Ratio $Q = 0.01 \cdot Q_0$ as a function of S and Ψ for different n. Curves 1, 2, and 3 correspond to the values 0, 1, and 5 for n.

convection. The position of the temperature maximum is determined by the expression $x = 1/S(1 + \Psi)$, and the position of the velocity extremum is determined by

$$x = \{-S(1+\Psi)(3+6\Psi) \pm \sqrt{S^2(1+\Psi)^2(3+6\Psi)^2+12}\}\ \frac{1}{6}.$$

It is seen from these equations that with an increase in Ψ the position of the maxima approaches the center x = 0, while the minimum in the velocity distribution now occurs at small S. Consequently, in this limiting case owing to the parameter Ψ the convection in the channel is already caused mainly by Joule heat release beginning with small values of S.

A numerical analysis of Eq. (10) is required in general. For a more graphic representation of the variation in temperature θ and velocity v profiles of θ and v are constructed in Fig. 1a, b for fixed values of S and n and different values of the parameter Ψ. It is seen from the graphs that the finite thickness and finite thermal conductivity has a marked effect on the temperature and velocity. With an increase in Ψ the Joule heat release builds up more and more, as a result of which the convection intensifies and the temperature and velocity profiles become more regular parabolas, i.e., the maximum of θ and v is displaced toward the center. This is well seen in Fig. 2a, b where the position of the maximum temperature and velocity is shown as a function of S for different Ψ at a fixed n. With an increase in Ψ a maximum in the temperature in the cavity, which does not exist in the case of mixed convection, already appears at small values of S.

The portion of the convection which is due to the allowance for the finite thickness and finite thermal conductivity of the walls is easily estimated quantitatively from the parameters S and n which provide for the appearance of a temperature maximum and the absence of return flow within the channel for different values of Ψ. Graphs of the values of n, S, and Ψ for which these conditions are satisfied were calculated and plotted. The curves in Fig. 3a correspond to a heat flux at the right wall equal to zero. In the region to the right of these curves the temperature nowhere can have a maximum. The curves in Fig. 3b correspond to the minimum velocity, equal to zero. To the left of these curves the velocity profile does not have a minimum, i.e., return flow. It is seen from the graphs that with an increase in Ψ the region where a temperature maximum exists in the cavity and there is no velocity minimum expands, and at small values of the parameter S the convection induced by the high-frequency current already predominates over natural convection: $n < 10$ for $0.15 \leq S < 10$ when $\Psi \geq 1$.

Thus, the allowance for the finite thickness and finite thermal conductivity of the walls leads to the fact that the convection and heat exchange become more intense. This is explained by the fact that heat is drawn from the walls into the surrounding medium less than in the case without conjugate thermal conditions.

In order to estimate up to what values of the parameter Ψ one can neglect the effect of the finite thickness and finite thermal conductivity of the walls on the convection and heat exchange it is appropriate to study the ratio of heat fluxes Q/Q_0 when Q is 1% of Q_0. Q_0 corresponds to Ψ = 0. Let us calculate the heat flux at the right wall

$$Q = -S\,\frac{\operatorname{sh} 2n + \sin 2n}{2n N_1(2n)} + \frac{1}{1+\Psi}\,.$$

The ratio Q/Q_0 as a function of Ψ and S for different values of the parameter n is shown in Fig. 4. In the region to the left of these curves the thermal conjugate boundary conditions do not play an important role. This region expands with an increase in n.

NOTATION

T and v	are the temperature and velocity;
λ, σ, ν, and β	are the coefficients of thermal and electrical conductivity, kinematic viscosity, and volumetric expansion;
ρ	is the density;
ω	is the angular frequency of current;
g	is the acceleration of force of gravity;
i	is the imaginary unit;
x	is the coordinate perpendicular to channel.

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